Nobel Laureate Amartya Sen has, for some time, argued persuasively that human development is best defined as the expansion of individual freedoms or capabilities. Freedom from poverty, illiteracy, infant malnutrition, and freedom to participate in political processes and economic exchange are common examples. Enhancing the capabilities of the members of society to live the kind of lives they have reason to value characterizes Sen’s development ideology. Only when each of society’s members have the capacity to avoid impediments to happiness like poverty, under-education, malnutrition, political unrest, and gender inequality can that society be referred to as developed.¹ Sen’s development focus, therefore, can be stated succinctly as the development of capabilities.

In partial fulfillment of the philosophy of Sen (as well as others sharing the same view), the United Nations Development Programme (UNDP) has maintained the Human Development Index (HDI) for the last decade, which assesses how well countries are doing in terms of non-income measures. The HDI is calculated as the simple average of life expectancy, education, and GDP indexes. In the UNDP’s words, the link between “...economic growth and human development...is by no means automatic,” which provides relevance for welfare comparisons based on indexes richer in information than GDP alone. Sen’s approach argues emphatically for the need to separate “means” and “ends”. For example, education often is viewed as a means to higher income. Alternatively, under the capabilities approach education is a laudable goal and therefore is itself a characteristic of development regardless of its monetary return. The freedom or capacity to live a long healthy life also is viewed as an ‘end goal’ or a characteristic of development and should therefore be pursued independent of its affect on income.

While the HDI is valuable, the simple averaging of the component indexes causes concern with the strict interpretation of the resultant rankings. For example, Ray (1998) states

The HDI might look scientific and the formulae used to create the final average might look complex, but that is no reason to accept the implicit weighting scheme that it uses, because it is just as ad hoc as any other. It cannot be otherwise. Nevertheless the HDI is one way to combine important development indicators, and for this reason it merits our attention. (Emphasis is original)

Though the components of the HDI are important aspects of human development, the weighting procedure is certainly unsettling. Is there any reason to suspect that education is exactly as important in determining human development as GDP or life expectancy? This postulate necessarily follows, however, if the simple average is the chosen method of combining the components. Also, to the extent one component index has a different variance than another equal weights seem unsatisfactory. Greater variability in one component index relative to another represents information that is unused or ignored in simple averaging. Further, economic and social variables inherently are interdependent. Cross-correlation between constituent indexes is important yet unused information unless multivariate methods that select weights accordingly are employed.

Consider the interdependence of the component indexes of the HDI. It is widely accepted that education (or human capital) is an important input in the production process. Mankiw, Romer, and Weil (1992) verify statistical significance between education levels and per capita GDP in cross-sectional samples of 98, 75, and 22 countries. Further, these authors

¹ Special thanks to Jaime Caliendo for helpful comments.
demonstrate that augmenting the standard Solow model with human capital explains an additional 20 percent of cross-country variation in per-capita income. Also, the general skill level of the workforce may yield substantial spillover effects to production (Lucas (1988)).\(^2\) Alternatively, Bils and Klenow (2000) show that current investment is education probably are a function of current and future income levels. Larger current income levels facilitate larger amounts of education, and larger expected income in the future provides incentives for current educational attainment. Additionally, the simple correlation coefficient between the GDP index and the education index is 0.77. Yet, according to Sen’s capabilities approach, education and income are both independent goals of economic development (education is desirable regardless of the return to skilled labor); as such, the correct weighting of the two components is unclear given the observed simultaneity.

Education and life expectancy also may be interdependent. A long life expectancy provides additional incentive to obtain education, since the term of the discounted stream of future wages is lengthened. In turn, education may yield higher life expectancy. An educated society might better understand the importance of preventative health care measures. Knowledge of the importance of regular physical check-ups and preventative examinations, and better understanding of the uses and capabilities of pharmaceuticals, are facilitated through education. Further, properly educated medical practitioners should be more productive in providing increased life expectancies to the public than under-educated practitioners. Perhaps most importantly, more education means *more physicians*, which improves life expectancy prospects in general. Lastly, the simple correlation coefficient between the life expectancy index and the education index is 0.82.

Arbitrary weighting procedures are unable to adequately capture the inherent interdependence of these component variables. Multivariate approaches, however, are able to incorporate this interdependence. If variability in the component indexes and interdependence among the components should be considered when selecting a weighting scheme, then the HDI appears at first glance to be insufficient.

In Sen’s work (Sen 1999), he acknowledges (though does not agree with) the critics of his development paradigm who argue that the arbitrariness of any chosen weighting procedure, for which non-pecuniary variables are to be weighted, prevents the capabilities approach from being operational. For example, T.N. Srinivasan criticizes the freedom or capabilities approach “where he worries about the varying importance of different capabilities and proposes the rejection of this approach in favor of the advantage of the real-income framework which includes an operational metric for weighing commodities—the metric of exchange.” Srinivasan’s critique reflects conventional skepticism surrounding the weighting procedure used by the HDI, where the components are given arbitrary weights, instead of, perhaps, weights that are consistent with the varying importance of the capabilities. The principal component analysis preformed below is able to, in some degree, overcome Srinivasan’s critique since it is a rigorous and reasonably objective method of combining non-income measures, which therefore serves as an alternative to subjective or ad hoc weighting schemes that potentially attach incorrect importance to the various capabilities.\(^3\)

The paper is organized as follows. Section 1 provides a brief overview of principal component analysis. Section 2 contains empirical results and comparison between the HDI and the first principal component development indexes. Very small differences between the two ranking systems are reported. It appears that the HDI is, in fact, less subjective than previously held. Section 3 examines the causes of the observed similarity between the ranking systems, and section 4 concludes the paper.

### 1. Principal component analysis

The first principal component index is defined as the linear combination of the constituent indexes with maximal variance. What is gained by ranking countries according to the principal component index? The goal of any development index is to separate or characterize different
countries according to the chosen measure of social wellbeing. Maximizing the variance of the linear combination is equivalent to maximizing information content in the subsequent index ranking. Since gauging ‘how well countries are doing’ relative to each other is the objective, arbitrary weighting schemes attach potentially incorrect weights to the component variables and may not be useful. On the other hand, the linear combination of the component series with maximal variance accounts for interdependence among component indexes, which ensures full utilization of the available information. In short, maximizing the variance serves to ‘spread-out’ the resultant rankings, which in turn serves to maximize information content and therefore helps to categorize or rank countries as effectively as possible.

Let the life expectancy index be \( X_1 \), the education index \( X_2 \), and the GDP index \( X_3 \). \( X \) is the horizontal concatenation of \( X_1, X_2, \text{ and } X_3 \). The first principal component, \( Y \), is the weighted average of \( X_1, X_2, \text{ and } X_3 \) with maximal variance. Thus, \( Y = Xa \), where \( a \) is a vector of optimal weights \( a = (a_1, a_2, a_3) \). The variance of \( Y \) is

\[
\text{var}(Y) = \text{var}(a_1 X_1 + a_2 X_2 + a_3 X_3), \quad \text{or}
\]

\[
\text{var}(Y) = \text{var}(a_1^2 s_{11} + a_2^2 s_{22} + a_3^2 s_{33} + 2a_1 a_2 s_{12} + 2a_1 a_3 s_{13} + 2a_2 a_3 s_{23})
\]

where \( s_{ij} \) represents the covariance between \( X_i \) and \( X_j \). It is evident from (2) that components with higher cross-country variability contribute more heavily to \( \text{var}(Y) \). Also, \( \text{var}(Y) \) is an increasing function of the interdependence of the component indexes. The component indexes with larger variances are given larger weights, and strong interdependence between two component indexes translates to larger weights for both indexes. Equation (2) can be written compactly as

\[
\text{var}(Y) = a' S a , \quad \text{where } S \text{ is the variance-covariance matrix of } X.
\]

The first principal component, \( Y \), maximizes \( \text{var}(Y) = a' S a \), subject to a normalization constraint \( a' a = 1 \). The lagrangian function is

\[
L = a' S a + \lambda (1 - a' a)
\]

Thus, \( a \) is selected to obey the following first order condition

\[
\frac{\partial L}{\partial a} = \frac{\partial (a' S a + \lambda (1 - a' a))}{\partial a} = 0
\]

which implies

\[
2(Sa - \lambda a) = 0 , \quad \text{or } (S - \lambda I)a = 0
\]

The multiplier can be interpreted as a characteristic root of the variance covariance matrix \( S \). The solution to the equation \( (S - \lambda I)a = 0 \) is the corresponding characteristic vector, \( a \). Thus, the roots are chosen such that \( |S - \lambda I| = 0 \). Multiple roots exist, but the largest is the one of interest. To see this, Pre-multiply \( (S - \lambda I)a = 0 \) by \( a' \)

\[
a' Sa - a' \lambda Ia = 0.
\]

Given the normalization \( a' a = 1 \), and given that \( \lambda \) is a scalar, (5) reduces to

\[
a' Sa = \lambda = \text{var}(Y).
\]

From (6), maximizing \( \text{var}(Y) \) is equivalent to selecting the largest characteristic root.\(^5\)

2. Empirical results and comparison with the HDI

Identification of the largest characteristic root, together with the normalization \( a' a = 1 \), gives a optimal weights with which to rank countries and compare to the HDI. The weights for
the life expectancy, education, and GDP indexes are 0.59, 0.60, and 0.55. Generally, constituent weights sum to one (see Ram (1982)); thus, scaling the elements of the characteristic vector accordingly yields 0.34, 0.34, and 0.32. Notice, the principal component weights are nearly identical to those used in the construction of the HDI where all weights are 0.33.

Re-ranking all 162 countries in the Human Development Report 2001 according to the first principal component index generates rankings that are largely consistent with those of the HDI (see appendix A for a listing of the HDI ranking versus the principal component ranking). The average change in rank (in absolute value terms) is equal to 0.85. For example, a country ranked 5 according to the HDI would on average be ranked by the principal component index between 4.15 and 5.85 (not more than 1 place away from its HDI ranking). Also, the rank correlation coefficient between the HDI and the new index is 0.99.

Despite the simplistic methodology, it appears that the HDI is a good method of combining the component indexes and should be viewed, perhaps, with less skepticism. But why is the first principal component index so similar to the HDI?

3. Explanation of similarity between indexes

Consider the variance-covariance matrix \( S \), which equals

\[
S = \begin{bmatrix}
0.038056 & 0.030756 & 0.030233 \\
0.030756 & 0.040006 & 0.029404 \\
0.030233 & 0.029404 & 0.03286
\end{bmatrix}.
\]

The elements of \( S \) are similar in magnitude. It appears that the covariances across and within the component indexes are not terribly different. It can be shown that if all the elements of \( S \) are equal, then the elements of the characteristic vector are all equal to one-third, which is equivalent to the simple average of the component indexes. Let \( S = \delta K \), where \( \delta \in \mathbb{R} \) and \( K \) is a 3x3 matrix whose elements are unity. Each characteristic root, \( \lambda \), satisfies the characteristic equation

\[
(\delta - \lambda)(\delta ^2 - \delta ^2) - (\delta - \lambda)(\delta ^2 - \delta ^2) + (\delta - \lambda)(\delta ^2 - \delta ^2) = 0.
\]

It is clear from (13) that any scalar will work. The characteristic vector, \( a \), solves the homogeneous equation \((S - \lambda I)a = 0\), wherein all the elements of \( a \) must be equivalent in order for this equation to hold. Thus, the simple average (equal weights of one-third to each component index) is equivalent to the multivariate analysis given above provided the elements of the variance-covariance matrix are identical. In the case of the variance-covariance matrix for the component indexes of the HDI, the elements are not exactly the same but are quite close. As such, the multivariate approach provides a ranking of countries that is not exactly the same as the HDI but is quite close.

4. Conclusion

This paper uses a multivariate technique called principal component analysis, which combines various measures of human development in an optimal fashion to create a development index. The first principal component represents an objective method of combining component indexes in a fashion that maximizes the information content of the resultant index. This multivariate technique accounts for differences in variances of component indexes as well as interdependence among the component indexes, both of which are necessary for creating an operational development index. Applying the first principal component analysis to the three indexes used in the HDI (life expectancy, education, GDP) yields a new index whose rankings are nearly identical to the original HDI.
We interpret this finding as theoretical support for the HDI ranking system as a metric of international human development. Since the simple average of the component indexes yields rankings roughly equivalent to a more complex multivariate technique that selects the weights optimally, this appears to be a case where little is lost in the simplistic method, and much is gained in terms straightforwardness. Indeed, while the strength of the HDI appears to lie in its easy comprehension, the weights used therein are consistent with multivariate techniques that generate weights optimally.

NOTES
1. This is only a partial list of the characteristics of a developed society according to Sen’s development philosophy.
2. Hall and Jones (1999) is another article highlighting the role of education in production.
3. Of course, Sen’s approach is much deeper than figuring out how to combine three measures of economic development in an optimal fashion. However, assessing optimal techniques for creating development indexes with non-income components is central to the application of Sen’s methodology.
4. This section follows the analysis provided in Morrison (1967) and Mardia, Kent and Bibby (1979).
5. The arbitrary normalization $a^*a = 1$ is for expositional ease only. Letting, for example, $a^* Sa = z$, where $z \in \mathbb{R}$ and $z \neq 1$, gives an equation analogous to (6): $a^* Sa = \lambda z = z \text{var}(Y)$. Still, the largest characteristic root of $S$ is chosen and the corresponding characteristic vector maintains proportionality. Thus, the ultimate ranking of countries is independent of the normalization of the characteristic vector.

REFERENCES