Global financial crisis, extreme interdependences, and contagion effects: The role of economic structure?

Riadh Aloui a, Mohamed Safouane Ben Aïssa a, Duc Khuong Nguyen b,*

a LAREQUAD & FSEG, University of Tunis El Manar, Boulevard du 7 novembre, B.P. 248, El Manar II, 2092 Tunis, Tunisia
b ISC Paris School of Management, Department of Economics, Finance and Law, 22, Boulevard du Fort de Vaux, 75848 Paris Cedex 17, France

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Abstract

The paper examines the extent of the current global crisis and the contagion effects it induces by conducting an empirical investigation of the extreme financial interdependences of some selected emerging markets with the US. Several copula functions that provide the necessary flexibility to capture the dynamic patterns of fat tail as well as linear and nonlinear interdependences are used to model the degree of cross-market linkages. Using daily return data from Brazil, Russia, India, China (BRIC) and the US, our empirical results show strong evidence of time-varying dependence between each of the BRIC markets and the US markets, but the dependency is stronger for commodity-price dependent markets than for finished-product export-oriented markets. We also observe high levels of dependence persistence for all market pairs during both bullish and bearish markets.

1. Introduction

The modern portfolio theory, relying on the seminal work of Markowitz (1952) and the underlying ideas of the Capital Asset Pricing Model (CAPM), posits that investors can improve the performance of their portfolios by allocating their investments into different classes of financial securities and industrial sectors that would not move together in the event of a valuable new information. Sub-perfectly correlated assets are thus particularly appropriate for adding to a diversified portfolio. Subsequently, Solnik (1974), among others, extends the domestic CAPM to an international context and suggests that diversifying internationally enables investors to reach higher efficient frontier than doing so domestically.

Empirically, Grubel (1968) examines the potential benefits of international diversification and shows the superiority of portfolios that are composed of both domestic assets and assets denominated in foreign currencies from 11 developed markets. These findings are then confirmed by other earlier studies where the analysis of market interdependence evolves both developed and developing countries (Levy and Sarnat, 1970; Errunza, 1977). The recent literature on measuring stock market comovements has been greatly stimulated by the globalization of capital markets around the world (Forbes and Rigobon, 2002; Gilmore et al., 2008; Abad et al., 2010). Based on a wide variety of methodologies, the majority of these works suggest that correlations of global stock returns have increased in the recent periods as a result of increasing financial integration, leading to lower diversification benefits especially in the longer term. More importantly, the level of market correlations varies over time.1

Modeling the comovement of stock market returns is, however, a challenging task. The main argument is that the conventional measure of market interdependence, known as the Pearson correlation coefficient, might not be a good indicator. It represents only the average of deviations from the mean without making any distinction between large and small returns, or between negative and positive returns (Poon et al., 2004). Consequently, the asymmetric correlation between financial markets in bear and bull periods as documented, for example, by Ang and Bekaert (2002), and Patton (2004) cannot be explained.2 The Pearson correlation estimate is further constructed on the basis of the hypothesis of a linear asso-

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1 See Longin and Solnik (1995) and references therein.
2 By asymmetric correlations, we mean that negative returns are more correlated than positive returns. This then suggests that financial markets tend to be more dependent in times of crisis. We also test this hypothesis within this paper.
ciation between the financial return series under consideration whereas their linkages may well take nonlinear causality forms (Beine et al., 2008). Other complications refer directly to stylized facts related to the distributional characteristics of stock market returns, in particular the departure from Gaussian distribution and tail dependence (or extreme comovement). Solutions for handling these problems include either the use of multivariate GARCH models with leptokurtic distributions which allow for both asymmetry and fat tails (Serban et al., 2007) or the use of multivariate extreme value theory and copula functions (Longin and Solnik, 2001; Pais and Stork, in press). Notice that the first modeling approach allows for capturing deviations from conditions of normality, while the last two approaches deal essentially with the extreme dependence structure of large (negative or positive) stock market returns, all in multivariate frameworks.

Since the investigation of dependence structure is crucial for risk management and portfolio diversification, this paper also focuses on the issue of interactions between financial markets. We are particularly interested in modeling the co-exceedances of stock market returns below or above a certain threshold. Our main objective is thus to look at the margins of stock market return distributions and test for both the degree and type of their dependence at extreme levels conditionally on the possibility of extreme financial events (e.g., financial turbulence, and crisis). Although we do not explicitly search for the determinants of cross-market financial dependence, we think that differences in the economic structure would be an important candidate for possible explanations and build our intuition on the basis of some prevailing economic indicators. For doing so, we combine the so-called conditional multivariate copula functions with extreme value theory as well as generalized auto-regressive conditional heteroscedasticity process (hereafter extreme value copula-based GARCH or EVC-GARCH models). In this nested setting, the GARCH models with possibly skewed and fat tailed return innovations are applied to filter the stock market returns and to draw their marginal distributions, while the multivariate dependence structure between markets is modeled by parametric family of extreme value copulas which are perfectly suitable for non-normal distributions and nonlinear dependencies. The model thus captures not only the tail dependence, but also the asymmetric tail dependence (i.e., the strength of market dependence may be different for extreme negative returns and for extreme positive returns). With regard to the methodological choice, our work is broadly similar to that of Jondeau and Rockinger (2006) who study the dynamics of dependency between four major stock markets, but it is more general in terms of GARCH specifications and copula functions. In addition, we demonstrate that portfolio managers will have an interest in employing EVC-GARCH models to estimate the value at risk in their internationally diversified portfolios during widespread market panics.

This present study also contributes to the related literature in that we provide a general framework for addressing the extent of extreme interdependencies and contagion effects between emerging and US markets, and among emerging markets themselves in the context of the 2007–2009 global financial crisis. This is important since knowing only the degree of time-varying comovement is actually not sufficient to make international investment decisions because stock market returns might exhibit common extreme variations. A number of past studies have reported the existence of significant linkages both between emerging and developed markets, and among emerging markets (e.g., Gallo and Otranto, 2005 for Asian emerging markets; Fujii, 2005 for Latin American emerging markets), but little is known about their extreme comovements. For instance, the work of De Melo Mendes (2005) investigates the asymmetric extreme dependence in daily log-returns for seven most important emerging markets using extreme value copula functions and shows some evidence of asymmetry in the joint co-exceedances for the majority of 21 pairs of markets considered. The cross-market tail dependence is also found to be stronger during bear market. Caillault and Guegan (2005) apply the Student and Archimedean copulas (Gumbel and Clayton) to daily data of three Asian emerging markets over the period from July 1987 to December 2002. They document that dependence structure is symmetric for Thailand–Indonesia, and Malaysia–Indonesia pairs. More recently, Hu (2008) examines the tail dependence between the Chinese stock market and the seven major developed markets by making use of the Normal and Generalized Joe–Clayton copulas. The author reports that time-varying dependence models are not always better than constant dependence models and that the upper tail dependence may be much higher than the lower tail dependence in some short periods. Note that our study differs from De Melo Mendes (2005) and Caillault and Guegan (2005) by allowing the marginal distributions of stock market returns to follow appropriate GARCH dynamics as well as the GARCH-in-Mean (GARCH-M) effects to control for the risk-return trade-off. Compared to Hu (2008), our GARCH specifications are more flexible since negative and positive shocks can affect the conditional variance differently. Further, the fact that we focus on the most important markets in the emerging universe (Brazil, Russia, India and China) with their differing economic systems allows us to shed light on the impact of economic structure on the extreme financial dependencies. Indeed, among our BRIC markets, Brazil and Russia can be viewed as commodity-price dependent countries, whereas India and China are finished-product export-oriented countries. The comparison of comovement levels among these markets is quite interesting because both commodity and finished-products prices have experienced lengthy swings during recent times.

Using daily returns on stock market indices over the period from March 22, 2004 to March 20, 2009, we mainly find that the GARCH-M specification which allows for asymmetric effects from negative and positive shocks is the most appropriate for the data, and that stock market volatility is highly persistent over time. With regard to copula modeling, the Gumbel extreme value copula appears to fit at best the tail dependence of the markets studied. More importantly, our results provide strong evidence of extremely negative and positive co-exceedances for all market pairs, but extreme comovement with the US is higher for commodity-price dependent markets than export-price

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3 The exceedance can be defined as the occurrence of an extreme return observation, i.e., a return value that is below (extreme negative return) or above (extreme positive return) a prespecified threshold of a financial market at a certain point in time (Teiletche and Xu, 2008). We then refer to the joint occurrence of exceedances in two particular markets at the same point in time as co-exceedance, which typically provides a measure of extreme comovement in financial markets. See also Christiansen and Ranaldo (2009) and Beine et al. (2010).

4 Copulas are functions that describe the dependencies between variables, and enable modeling their joint distribution when only marginal distributions are known. The main applications of copulas in finance can be found in Cherubini et al. (2004).

5 Jondeau and Rockinger (2006) focus on the US, the UK, German and French stock markets represented respectively by the S & P500, FTSE, DAX and CAC40 indices.

6 In a recent study, Rodríguez (2007) uses a copula approach with regime-switching parameters to model the dependence of daily returns from five Asian emerging markets, and four Latin American emerging markets during the 1997–1998 Asian crisis and the 1994–1995 Mexican crisis. The author finds evidence of changing dependence during times of crisis. However, the methodology adopted basically consists of fitting the copulas-based regime-switching ARCH models to the whole distribution of market returns, which is not the focus of our paper albeit the objective is somewhat similar.

7 Hu (2008) considers the following developed markets: France, Germany, Hong Kong, Japan, the United Kingdom, and the United States. Data are daily stock market indices covering the period from January 1991 to December 2007. A comprehensive description of the generalized Joe–Clayton copula can be found in Patton (2006).
dependent markets. Within the universe of BRIC markets, the results indicate that they are more dependent in the bull markets than in the bear markets.

The remainder of this paper is organized as follows: Section 2 presents the theoretical background of the copula functions and shows how they can be applied to study the extreme comovements between the BRIC markets and the US, especially over the 2007–2009 period of the global financial crisis. In Section 3, the empirical results are reported and interpreted with reference to the economic structure of the emerging markets considered. We provide summary of our conclusions in Section 4.

2. Copula functions and their applications

Copulas are functions that link multivariate distributions to their univariate marginal functions. A good introduction to copula models and their fundamental properties can be found in Joe (1997) and Nelsen (1999). Formally, we refer to the following definition:

Definition 1. A d-dimensional copula is a multivariate distribution function \( C \) with standard uniform marginal distributions.

**Theorem 1** (Sklar’s theorem). Let \( X_1, \ldots, X_d \) be random variables with marginal distribution \( F_1, \ldots, F_d \) and joint distribution \( H \), then there exists a copula \( C: [0,1]^d \rightarrow [0,1] \) such that:

\[
H(X_1, \ldots, X_d) = C(F_1(X_1), \ldots, F_d(X_d)).
\]

Conversely if \( C \) is a copula and \( F_1, \ldots, F_d \) are distribution functions, then the function \( H \) defined above is a joint distribution with margins \( F_1, \ldots, F_d \).

Therefore copulas functions provide a way to create distributions that model correlated multivariate data. Using a copula, one can construct a multivariate distribution by specifying marginal univariate distributions, and then choose a copula to detect how they can be applied to study the extreme comovements between the BRIC markets and the US, especially over the 2007–2009 period of the global financial crisis. In Section 3, the empirical results are reported and interpreted with reference to the economic structure of the emerging markets considered. We provide summary of our conclusions in Section 4.

(i) We first test the presence of ARCH effects in raw returns using the ARCH LM test. Various GARCH specifications that allow for the leverage effect are estimated and compared using the usual information criteria such as AIC, BIC and Log-lik statistics. We choose the GARCH-M model as it gives the best fit. This model extends the basic GARCH model by allowing the conditional mean to depend directly on the conditional variance. The conditional variance specification considered allows for a leverage effect, i.e. it may respond differently to previous negative and positive innovations. Instead of assuming normal distributions for the errors, we use the Student-t distribution to capture the fat tails usually present in the model’s residuals. The GARCH-M model may be expressed as:

\[
y_t = c + \lambda \sigma_t^2 + \epsilon_t,
\]

\[
\sigma_t^2 = \omega + \alpha (|\epsilon_{t-1}| - \gamma \sigma_{t-1})^2 + \beta \sigma_{t-1}^2,
\]

where \( c \) is the mean of \( y_t \) and \( \epsilon_t \) is the error term which follows a Student-t distribution with \( v \) degrees of freedom. A positive GARCH-in-Mean term \( \lambda \) implies that higher risk is positively associated with higher return. The conditional variance equation depends upon both the lagged conditional standard deviations and the lagged absolute innovations. Here the GARCH model works like a filter in order to remove any serial dependency from the returns.

(ii) We consider the innovations computed in step 1 and we fit the generalized Pareto distribution (GPD) to the excess losses over a high threshold. We note that in the extreme value theory (EVT) tail of any statistical distribution can be modeled by the GPD. The use of EVT is of great importance for emerging markets since they are significantly influenced by extreme returns (Harvey, 1995). The main difference between emerging and developed markets resides in the tail of the empirical distribution produced by extreme events. More specifically, stock returns from emerging markets have significantly fatter tails than stock returns from developed markets.

(iii) The uniform variates are obtained by plugging the GPD parameter estimates into the GPD distribution function and the following selected copula models belonging to the extreme value copula family (the Gumbel, Galambos, and Husler-Reiss copulas) are fitted.

The Gumbel Copula

Gumbel (1960) is probably the best-known extreme value copula. It is an asymmetric copula with higher probability concentrated in the right tail. By contrast, the Gumbel copula retains a strong relationship even for the higher values of the density function in the upper right corner. It is given by

\[
C(u, v) = \exp\left(-\left(-\ln u\right)^\delta + \left(-\ln v\right)^\delta\right), \quad \delta \geq 1.
\]

The parameter \( \delta \) controls the dependence between the variables. When \( \delta = 1 \) there is independence and when \( \delta \rightarrow +\infty \), there is perfect dependence. The coefficient of upper tail dependence for this copula is

\[
\lambda_u = 2 - 2^{1/\delta}.
\]

The Galambos copula

Galambos (1975) is

\[
C(u, v) = uv \exp\left(-\left(-\ln u\right)^\delta + \left(-\ln v\right)^\delta\right), \quad 0 \leq \delta < \infty.
\]

The Husler–Reiss Copula

introduced by Husler and Reiss (1987) has the following form:

\[
C(u, v) = \exp\left(-\left(-\ln u\right)^\delta + \left(-\ln v\right)^\delta\right), \quad 0 \leq \delta < \infty.
\]

\[\text{See Joe (1997) for complete references for Gumbel (1960), Galambos (1975), and Husler and Reiss (1987).}\]
where $\phi$ is a CDF of a standard Gaussian distribution, $\tilde{u} = -\ln(u)$ and $\tilde{v} = -\ln(v)$. Fig. 1 shows the contour plots of the selected copula models. These plots are very informative about the dependence properties of the copulas. For this reason, one often uses contour plots to visualize differences between various copulas and possibly to assist in choosing appropriate copula functions.

In order to fit copulas to our data, we use the method proposed by Joe and Xu (1996) called inference functions for margins (IFM). This method first determines the estimate of the margin parameters by making an estimate of the univariate marginal distributions and then the parameters of the copula. The IFM method has the advantage of solving the maximization problem for cases of high dimensional distributions.

Two goodness-of-fit tests are used to compare copula models. These tests, qualified by Genest et al. (2009) as “blanket” tests, are based on empirical copula and on Kendall’s process.

Specifically, the statistics considered use both the Cramér–Von Mises distances as

$$S_n = n \int (C_n(u, v) - C_{h_n}(u, v))^2 dC_n(u, v)$$

and

$$T_n = n \int (K_n(w) - K_{h_n}(w))^2 dK_n(w).$$

The first statistics $S_n$ measures how close the fitted copula $C_{h_n}$ is from the empirical copula $C_n$, while the second statistics $T_n$ measures the distance between an empirical distribution $K_n$ and a parametric estimation $K_{h_n}$ of $K$. The null hypothesis that the copula $C$ belongs to a class $C_0$ is rejected for high values of the computed test statistics. The $p$-values associated with the tests are computed using a parametric bootstrap procedure and the validity of such an approach is established in Genest and Rémillard (2008).

3. Empirical results

3.1. Data and stochastic properties

We empirically investigate the interaction between various stock market indices. Specifically, the data consist of five indices...
representing four emerging economies, namely Brazil, Russia, India, and China, together with the US market index. All data are the MSCI total return indices expressed in US dollars on a daily basis from March 22, 2004 to March 20, 2009. The returns are calculated by taking the log difference of the stock prices on two consecutive trading days, yielding a total of 1304 observations.

To assess the distributional characteristics and stochastic properties of the return data, we must first examine some descriptive statistics reported in Table 1. The reported statistics show that all the data series are negatively skewed and exhibit excess kurtosis, which indicates evidence that the returns are not normally distributed. The Jarque–Bera statistics are highly significant for all return series and just confirm that an assumption of normality is unrealistic. Moreover, the Ljung-Box statistics (lags 12) suggest the existence of serial correlations in all the data series. Both the Ljung-Box statistics for the squared returns and the ARCH LM test are highly significant, which indicates the presence of ARCH effects in all the series.

Fig. 2 illustrates the variation of stock returns in five markets. From the graph, we can see that the stock prices were fairly stable during the period from March 2004 to the third quarter of 2008. After this date all return series displayed more instability due in particular to the global financial crisis.

Table 2 reports the unconditional correlations for all return series. As expected, there is a positive correlation between the US and BRIC markets. The highest correlation is between the US and Brazil (0.63) and the lowest one is between US and China (0.20). The same is true for emerging markets, although the China–India and the Russia–Brazil markets are more correlated than other BRIC markets with correlations of 0.57 and 0.53, respectively.

**Table 1**

<table>
<thead>
<tr>
<th></th>
<th>Brazil</th>
<th>Russia</th>
<th>India</th>
<th>China</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>-0.183</td>
<td>-0.255</td>
<td>-0.120</td>
<td>-0.128</td>
<td>-0.095</td>
</tr>
<tr>
<td>Max</td>
<td>0.166</td>
<td>0.239</td>
<td>0.088</td>
<td>0.140</td>
<td>0.110</td>
</tr>
<tr>
<td>Mean</td>
<td>6.783e-004</td>
<td>-2.161e-004</td>
<td>2.627e-004</td>
<td>3.607e-004</td>
<td>-2.59e-004</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>2.68e-002</td>
<td>2.813e-002</td>
<td>2.043e-002</td>
<td>2.168e-002</td>
<td>1.407e-002</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.431</td>
<td>-0.513</td>
<td>-0.640</td>
<td>-0.049</td>
<td>-0.348</td>
</tr>
<tr>
<td>Ex. kurtosis</td>
<td>7.836</td>
<td>17.28</td>
<td>4.573</td>
<td>6.404</td>
<td>13.198</td>
</tr>
<tr>
<td>Q (12)</td>
<td>26.346*</td>
<td>78.254*</td>
<td>55.870*</td>
<td>18.820***</td>
<td>63.609*</td>
</tr>
<tr>
<td>Q2(12)</td>
<td>1584.60*</td>
<td>688.78*</td>
<td>709.44*</td>
<td>1080.83*</td>
<td>1650.27*</td>
</tr>
<tr>
<td>J–B</td>
<td>3348.016*</td>
<td>16163.85*</td>
<td>1214.66*</td>
<td>2209.21*</td>
<td>9412.38*</td>
</tr>
<tr>
<td>ARCH(12)</td>
<td>505.211*</td>
<td>266.090*</td>
<td>238.288*</td>
<td>354.732*</td>
<td>454.186*</td>
</tr>
</tbody>
</table>

Notes: The table displays summary statistics for daily returns for the five countries. The sample period is from March 22, 2004 to March 20, 2009. Q(12) and Q2(12) are the Jarque–Bera statistics for serial correlation in returns and squared returns for order 12. ARCH is the Lagrange multiplier test for autoregressive conditional heteroskedasticity.

* The rejection of the null hypotheses of no autocorrelation, normality and homoscedasticity at the 1% levels of significance respectively for statistical tests.

** The rejection of the null hypotheses of no autocorrelation, normality and homoscedasticity at the 5% levels of significance respectively for statistical tests.

*** The rejection of the null hypotheses of no autocorrelation, normality and homoscedasticity at the 10% levels of significance respectively for statistical tests.

Fig. 2. Time paths of daily returns for Brazil, Russia, India, China and the US.
The estimated changes. Use of the asymmetric GARCH-M model seems to be justi-
tent, i.e. large changes in the conditional variance are followed by
can be indeed selected according to these statistics. The results of the
specifications of the GARCH models. The GARCH-M volatility mod-
relates to the value of their tail dependence coefficient. All the pairs con-
sidered are mutually dependent during bear and bull markets. Most of the symmetric fit are based on the Gumbel copula which
gives the best fit in most cases. We first note that the pair Bra-
zil–US is the strongest tail dependent pair for both positive and
co-exceedances. The second and the third position are occupied by the Russia–US and India–US pairs; the China–US mar-
stock markets. To examine and measure market risk, the most commonly used tech-
the extreme value theory is less likely to be violated by the filtered
series. Fitting the GPD to the filtered returns requires specification of the lower and upper thresholds. Following De Melo Mendes (2005), we set the threshold values such that 10% of the residuals are reserved for the lower left and the upper right quadrants (i.e., the empirical 10% and 90% quantiles in each margin). In what fol-
We propose the use of copula to quantify the risk of three equally-
portfolio value of given confidence level over a given time period.
the prices of many instruments and across many markets. To
estimation (QMLE) method a GARCH model for the return data. Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC) and the log-likelihood function are used to compare various specifications of the GARCH models. The GARCH-M volatility mod-
el is indeed selected according to these statistics. The results of the
GARCH-M fitting are reported in Table 3. For all return series, we
Note that the parameter \( \theta \) is close to 0.9 with an extremely signifi-
can be estimated upon request addressed directly to the corresponding author.

### Table 2

<table>
<thead>
<tr>
<th></th>
<th>Brazil</th>
<th>Russia</th>
<th>India</th>
<th>China</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil</td>
<td>1</td>
<td>0.537</td>
<td>0.363</td>
<td>0.424</td>
<td>0.639</td>
</tr>
<tr>
<td>Russia</td>
<td>1</td>
<td>0.391</td>
<td>0.449</td>
<td>0.306</td>
<td></td>
</tr>
<tr>
<td>India</td>
<td>1</td>
<td>0.570</td>
<td>0.254</td>
<td></td>
<td></td>
</tr>
<tr>
<td>China</td>
<td>1</td>
<td>0.206</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: This table gives the unconditional correlation between daily returns of BRIC and US markets.

### Table 3

<table>
<thead>
<tr>
<th></th>
<th>Brazil</th>
<th>Russia</th>
<th>India</th>
<th>China</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>0.11e-2</td>
<td>0.41e-3</td>
<td>0.34e-2</td>
<td>2.626e-3</td>
<td>2.311e-3</td>
</tr>
<tr>
<td>( \beta )</td>
<td>-0.182</td>
<td>-0.344</td>
<td>-3.988</td>
<td>-2.173</td>
<td>-1.349</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.17e-4</td>
<td>0.145e-4</td>
<td>0.10e-4</td>
<td>4.025e-6</td>
<td>1.147e-5</td>
</tr>
<tr>
<td>( \phi )</td>
<td>0.070</td>
<td>0.140</td>
<td>0.130</td>
<td>8.923e-2</td>
<td>0.020</td>
</tr>
<tr>
<td>( \omega )</td>
<td>0.868</td>
<td>0.837</td>
<td>0.817</td>
<td>0.897</td>
<td>0.928</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>2.377e-2</td>
<td>2.493e-2</td>
<td>2.616e-2</td>
<td>1.638e-2</td>
<td>1.531e-2</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.540</td>
<td>0.298</td>
<td>0.437</td>
<td>0.232</td>
<td>0.992</td>
</tr>
<tr>
<td>( \theta )</td>
<td>2.60e-1</td>
<td>7.679e-2</td>
<td>1.305e-1</td>
<td>8.817e-2</td>
<td>4.093e+1</td>
</tr>
</tbody>
</table>

Notes: The table summarizes the GARCH-M estimation results. The values between brackets represent the standard error of the parameters.

* Significance at the 1% levels.
** Significance at the 5% levels.
*** Significance at the 10% levels.

### 3.2. Estimation results

In the first step we fit by using the quasi-maximum likelihood estimation (QMLE) method a GARCH model for the return data. Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC) and the log-likelihood function are used to compare various specifications of the GARCH models. The GARCH-M volatility mod-
el is indeed selected according to these statistics. The results of the
GARCH-M fitting are reported in Table 3. For all return series, we
note that the parameter \( \beta \) is close to 0.9 with an extremely signifi-
can be estimated upon request addressed directly to the corresponding author.

In the second step we extract the filtered residuals from each returns series with an asymmetric GARCH-M model, and then we
construct the marginal of each series using the empirical CDF for the interior and the GPD estimates for the upper and lower tails. The advantage of this method is that the i.i.d assumption behind

#### 3.3. Estimating the value at risk

Financial institutions are exposed to risk from movements in the prices of many instruments and across many markets. To examine and measure market risk, the most commonly used tech-
nique is the value at risk (VaR), defined as the maximum loss in a
portfolio value of given confidence level over a given time period.

In Table 4 we also report the results for the joint losses and joint gains for the following pairs: Brazil–Russia, Brazil–India, Brazil–China, Russia–India, Russia–China and India–China. We plot a scatter-
pplot of the considered markets pairs in Fig. 6. We observe that
the first three positions are occupied by India–China, Brazil–Russia and Russia–China. For Brazil–Russia, Brazil–China, and Russia–China
the dependence in the left lower tail is smaller than the dependence
in the right tail, i.e. these markets tend to comove more closely during bull markets than during bear markets.
In order to compute risk measures such as value at risk and expected shortfall, we have to use Monte Carlo simulations because analytical methods exist only for a multivariate normal distribution (i.e., a Gaussian copula). When copula functions are used, it is relatively easy to construct and simulate random scenarios from the joint distribution of $X$ and $Y$ based on any choice of marginals and any type of dependence structure.

Our strategy consists of first simulating dependent uniform variates from the estimated copula model and transforming them into standardized residuals by inverting the semi-parametric marginal CDF of each index. We then consider the simulated standardized residuals and calculate the returns by reintroducing the GARCH volatility and the conditional mean term observed in the original return series. Finally, given the simulated return series, for each pair $(x_i, y_i)$ we compute the value of the global portfolio $R$. The VaR for a given level $q$ is simply the $100q$th percentile of the loss distribution, expressed analytically as

$$\text{VaR}_q = F_{-R}^{-1}(q).$$

Consequently, the expected shortfall, defined as the expected loss size given that VaR$_q$ is exceeded, is given by

$$ES = E[-R | -R > \text{VaR}_q].$$

In order to assess the accuracy of the VaR estimates, a backtest for the 95%, 99%, and 99.5% VaR estimates was applied. First, at time $t_0$ we estimate the whole model (GARCH + GPD + Copula) using data only up to this time. Then by simulating innovations from the copula we obtain an estimate of the portfolio distribution and estimate the VaR using model (1). This procedure can be repeated until the last observation and we compare the estimated VaR with the actual next-day value change in the portfolio. The whole process is repeated only once in every 50 observations owing to the computational cost of this procedure and because we did not expect to see large modifications in the estimated model when only a fraction of the observations is modified. However, at each new observation the VaR estimates are modified because of changes in the GARCH volatility and the conditional mean. If selected models are well suited for calculating VaR, the numbers of exceedances from these models should be then close to the expected numbers. Notice that the expected number of exceedances at $(1 - \alpha)$ confidence level over a backtesting period of $N$ days is equal to $\alpha \cdot N$.

We started by estimating the model using a window of 750 observations. Then we simulate 5000 values of the standardized residuals, estimate the VaR and count the number of the losses that exceed these estimated VaR values. At observations $t = 800, 850, \ldots, 1300$, we re-estimate the model and repeat the whole procedure.

We also estimate the VaR using two other approaches: the variance–covariance (also known as analytical) and the historical simulation methods. The first approach estimates the VaR assuming that the joint distribution of the portfolio returns is normal. The VaR for a given level $q$ is computed as follows:

$$\text{VaR}_q = \mu - z_{\alpha} \cdot \sigma,$$

where $\mu$ and $\sigma$ are the mean and the standard deviation of the portfolio returns, and $z_{\alpha}$ denotes the $(1 - \alpha)$ quantile of the standard normal distribution for our chosen confidence level. The main advantage of this method is its appealing simplicity. However, it suffers from several drawbacks. Among these, there is the fact that it gives a poor description of extreme tail events because it assumes that the risk factors are normally distributed. Also, the parametric method inadequately measures the risk of nonlinear instruments, such as options or mortgages.

The second approach considered is non-parametric, which means that it does not require any distributional assumptions for the probability distribution. The historical simulation estimates the VaR by means of ordered Loss–Profit observations. More generally, assume that we have $N$ sorted simulated Loss–Profit observations, then the VaR at the desired confidence level $(1 - \alpha)$...
corresponds to the \( x \cdot N \) th order statistics of the sample. Like the parametric method, historical simulation may be very easy to implement, but it suffers from a major drawback related to its dependence on a particular historical moving window. So when running this method immediately after a special crisis, this event will naturally be omitted from the window and the estimated VaR may change abruptly from one day to another.

In case of the variance–covariance and historical simulation methods, the model parameters were updated for every observation. The results for the backtesting are reported in Table 5. We
Lopez and Saidenberg (2001) report some of the difficulties in evaluating the quality of VaR estimates produced by these models. Effectively, the number of exceedances from the copula-based GARCH model is closer to the expected number of exceedances than for alternative models, while both statistic tests (Panel B) are satisfactory for the Brazil–US and Brazil–Russia pairs and coherent with the results obtained from Kupiec’s test.

Table 5 reports the results from backtesting procedure as well as the corresponding p-values of the POF statistics to test the null hypothesis of correct unconditional coverage and the DQCC statistics to test the null hypothesis of conditional efficiency, using the 95% VaR confidence level and a sample size of 250 observations. According to Kupiec backtest, the EVC-GARCH model performs well for all considered portfolio. The reported p-values of the DQ test are satisfactory for the Brazil–US and Brazil–Russia pairs and coherent with the results obtained from Kupiec’s test. The only exception is for the India–China portfolio, since the p-value of the DQ test is inferior to the conventional significance level.

Overall, the findings lead to not reject the accuracy of our VaR model. Effectively, the number of exceedances from the copula-based GARCH model (Panel A) is closer to the expected number than for alternative models, while both statistic tests (Panel B)
are, in almost all cases, unable to reject the null hypothesis we examine at the conventional significance levels.

3.4. The effects of economic structure on financial comovements

Among emerging markets, the BRIC group of markets represents a different and dynamic set of investment opportunities. On the one hand, their economic rationales are straightforwardly linked to their size and their contributions to global economic growth. More precisely, a combination of large human capital, competitive work force, access to natural resources, and a sustainable revitalization of internal demand has substantially increased the role of the BRIC economies in the global economy. Today, they collectively account for nearly 30% of global output, and only China and India contribute about 1.16% and 0.41% to the global GDP growth according to the IMF’s World Economic Outlook Report of April 2008. Growth projections for these economies in the coming years are also substantially above the average growth of the world and developed economies, leading economists and experts to expect that, based on their potential of internal demand expansion and spending power, they could provide a cushion against slower growth in the global economy.

On the other hand, the rationales for equity and foreign direct investments rely particularly on the specificities of the BRIC markets, which can be considered as traditional emerging markets, compared for example to those of the most advanced emerging markets (e.g., Mexico, South Korea, and Taiwan). The BRIC markets have experienced spectacular increase in size as measured by ratios of stock market capitalization to GDP over the recent years, are less correlated with developed markets, and display higher idiosyncratic risk due to the low level of their market sensitivities to global factors. Additionally, it is worth noting that growing internal markets are helping the BRIC countries to substantially reduce their strong dependence on external markets such as the European Union, and the US, and thus their vulnerability to external shocks. As a result, this creates incentives for investors to consider dedicated strategies of asset allocations to the BRIC markets.

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Although the BRIC markets have many features in common as discussed previously, the empirical evidence we report here indicates that they do not behave similarly in regard to their financial linkages with the US. If we refer to Table 4, we observe that extreme financial dependency on the US during the 2007–2009 global financial crisis is much stronger for Brazil and Russia than for

10 The contribution of the US, Euro area, Japan and other developed countries reached only 1.53%, while other developing countries accounted for 1.76%.
China and India; meanwhile the latter have established important trade links with the world economy. Table 6 shows that both China and India have a high degree of economic openness with trade to GDP ratios of 71.3% and 44.9%. In particular, the shares of China's exports and imports in the world total trade activities are 9.1% and 6.9%, respectively. A careful inspection of the trade profiles of the markets studied reveals that Brazil and Russia are more dependent on the revenues from exports of commodities products (51.8% and 79.5% of the respective economies’ total exports), whereas the economic performance of China and India depends greatly on exports of manufactured products (93.4% and 64.0% of the respective economy's total exports). In other words, countries with higher sensitivity to commodity-price changes tend to comove closely with the US in both bull and bear markets. Our results contrast, to some extent, the findings of Teiletche and Xu (2008) according to which trade linkage variables such as import demand and trade competition appear as the most important determinants of extreme dependence between financial markets.

Taken together, we think that the heterogeneity of the BRICs’ economic structure and especially the trade profiles could, to this extent, be a prevailing explanatory factor for the cross-market extreme interdependences. It would be, for future research, interesting to quantify the impact of different types of economic structure on market comovement by running cross-sectional studies. Of course, an equal attention should be given to financial characteristics of sample markets since they also constitute important channels for financial contagion. Extreme financial dependence is for example expected to be stronger for countries with deeper and more liquid financial markets. However, one can remark that financial variables might be less relevant in view of the two market indicators we show in the lower part of Table 6. For instance, stock markets in both China and India are less dependent on the US shocks, while they have higher ratios of market capitalization and liquidity to GDP.

Notes: This table presents the trade profiles and some financial characteristics of BRIC markets in 2007. The total exports of commodities products are equal to the sum of the exports of agricultural, and fuel and mining products. The trade to GDP ratios represent the average of trade to GDP ratios over the period 2005–2007. Relative stock market size refers to the market capitalization of listed companies in percentage of GDP. Turnover ratio corresponds to the value of shares traded in percentage of market capitalization. Data are expressed in millions of US dollar and obtained from World Trade Organization statistics database and World Development Indicators database.

11 The total commodities exports include the exports of agricultural, and fuels and mining products. The interested readers can refer to the WTO’s statistics database for a complete list of the highest-value exported products for BRIC countries.

4. Concluding remarks

Studies of the transmission of return and volatility shocks from one market to another as well as studies of the cross-market correlations are essential in finance, because they have many implications for international asset pricing and portfolio allocation. Indeed, a higher degree of comovement (or correlation) between markets would reduce the diversification benefits and imply that at least a partially integrated asset pricing model is appropriate for modeling the risk-return profile of the assets issued by the considered countries. With the advent of the current global financial crisis in the aftermath of the US housing market failure, not only academic researchers but also investors and policymakers have shifted their attention to the extreme dependence structure of financial markets. This is explained by their shared concerns regarding the harmful consequences of contagion effects.

This paper employs a multivariate copula approach to examine the extreme comovement for a sample composed of four emerging markets and the US markets during the 2004–2009 period. The use of this method is advantageous in that it satisfactorily captures the tail dependencies between the markets studied, when univariate distributions are complicated and cannot be easily extended into a multivariate analysis (Jondeau and Rockinger, 2006). The copula functions also provide an interesting alternative to the traditional assumption of jointly normal distribution series, which appears to be unrealistic given the stochastic properties of the return data.

We first provide evidence of the superiority of a Student-t GARCH-in-Mean specification which allows for leverage effects in explaining the time-variations of daily returns on stock market indices. When calibrating several well-known copulas based on the marginal distributions of the filtered returns from the selected GARCH model, we find evidence of extreme comovement for all market pairs both in the left (bearish markets) and right tails (bullish markets). Further, the results suggest that dependency on the US is higher and more persistent for Brazil and Russia – countries which are highly dependent on commodity prices – than for China and India whose economic growth is largely influenced by finished-products export-price levels. Finally, the extreme dependence between emerging market pairs is found to be generally smaller in bearish markets than in bullish markets, which might indicate a low probability of simultaneous crashes. As a practical exercise to check the usefulness of the copula models developed in the paper, we estimate the value at risk for three equally-weighted portfolios for three couples of countries exhibiting tighter extreme comovements over the study period. The results indicate that the extreme value copula-based VaR model outperforms the analytical approach and historical simulation method. Undoubtedly, copula models fit best financial data during widespread market panics and frictions, where the approximations of the usual probability distributions are likely to be strongly biased.

Given the increasing interest in detecting potential gains from international portfolio diversification in a globalized context, further investigation of stock market relationships is needed. Future extensions of this work could focus on an explicit explanation of extreme financial interdependence, usingcountry-specific fundamentals.

References


